Muon polarisation in the neutrino factory

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Abstract

Muon polarisation leads to interesting effects and could be useful in enhancing one flavour of neutrinos over the other. Preserving it leads to some design constraints on the accelerators, on the beam energy, and on the storage ring. The natural polarization of 27% or more can be preserved in the capture cooling and acceleration process without too much difficulty. Preserving this level against the depolarising effect of energy spread in the storage ring requires either i) a bow-tie geometry (no spin precession) or ii) for a standard ring with spin precession, an RF bunching system that induces synchrotron oscillations with a synchrotron tune $Q_s \simeq \sigma(E)/E$. In the case of a standard ring, spin precession will normally reduce the effective longitudinal polarisation from 27% to 18%, unless the beam energy is chosen to be exactly (i.e. within 2.10^{-4}) $E = 45.311$ GeV, or multiple thereof. Odd multiples of this number provide the additional advantage of automatic polarisation reversal turn after turn. These constraints do not apply for bow-ties, which however require spin manipulation devices prior to injection in the muon decay ring. The polarisation figure of merit being the product of the neutrino flux by the square of the polarisation, improvements on the natural muon polarisation at the source have to be weighted accordingly against the loss of flux.

1 Muon polarisation at the source

Muons are naturally polarised in pion decay. In the $\pi^+ \rightarrow \mu^+\nu_\mu$ rest frame both ν_μ and μ^+ are left-handed. The muon polarisation in the laboratory is obtained by appropriate rotation and boost of the spin 4-vector (0,0,0,-1) corresponding to the left-handed μ^+ in its rest frame. The resulting average helicity of the muon, or longitudinal polarisation, is reduced from -100% (for a pion at rest) to $< h > \simeq -P^*/E^* = -27\%$ for pions above 200-300 MeV momentum. For a pion of given momentum, muon polarisation is correlated with muon momentum. Therefore, as described in [1], a monochromatisation of the pions followed by i) a drift to separate muons of different momenta and ii) collection in successive RF buckets, should allow separation in different

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bunches of muons of different polarisations. This does not change the *average* polarisation, but the *figure of merit* for polarisation asymmetry experiments [2] is $\Phi \times \mathcal{P}^2$, where Φ is the neutrino flux and $\mathcal{P}$ the polarisation. The *effective* polarisation is then the flux-weighted average $\sqrt{<\mathcal{P}^2>}$, and can be increased to 40% [1]. Polarisation filtering is therefore quite useful, as long as it does not introduce too much loss of intensity.

The muon spin precesses in electric and magnetic fields that are present during cooling and acceleration, but the muon spin tune ν – the number of additional spin precessions happening when the muon makes a complete turn – is very low:

$$\nu = a_\mu \gamma = \frac{g_\mu - 2}{2} \frac{\mathrm{E_{beam}}}{m_\mu} = \frac{\mathrm{E_{beam}}(\mathrm{GeV})}{90.6223(6)} .$$

It has been estimated [3] for the muon collider that a longitudinal polarisation of about 20% will resist all muon handling up to the injection into the collider. For the neutrino factory, the amount of cooling that is necessary, and probably the corresponding depolarisation, is much reduced. One therefore expects that an average muon polarisation of a least 27% will be available at the output of the capture and cooling channel, and maybe even an effective polarisation of 40%.

It is easy to obtain muon longitudinal polarisation if the acceleration and injection chain is situated in a plane. The total spin precession has to be a multiple k of π, leading to the constraint:

$$\sum_i E_i/E_0 \times \theta_i = k\pi \tag{1}$$

where $E_0 = 90.6223(6) GeV$, the sum runs over all bends by angles θ_i that the muons undergo with energies E_i. The angles are signed, therefore dog-bone-type recirculators, in which the two end-bends can be of opposite directions, could provide useful flexibility. This condition 1 will be necessary for some operation modes but not all, as described below.

2 Spin precession in the storage ring

In the storage ring, spin precession and depolarisation will take place. First, the spin will precess, so that its orientation will be different turn after turn. Unless the spin tune is exactly a multiple of 1/2, and the spin orientation was exactly longitudinal at injection, some longitudinal polarisation will be lost. The loss in effective polarisation for any energy besides integers and half integers is typically 30%, the average $\sqrt{<\mathcal{P}^2>}$ being 18% instead of 27%.

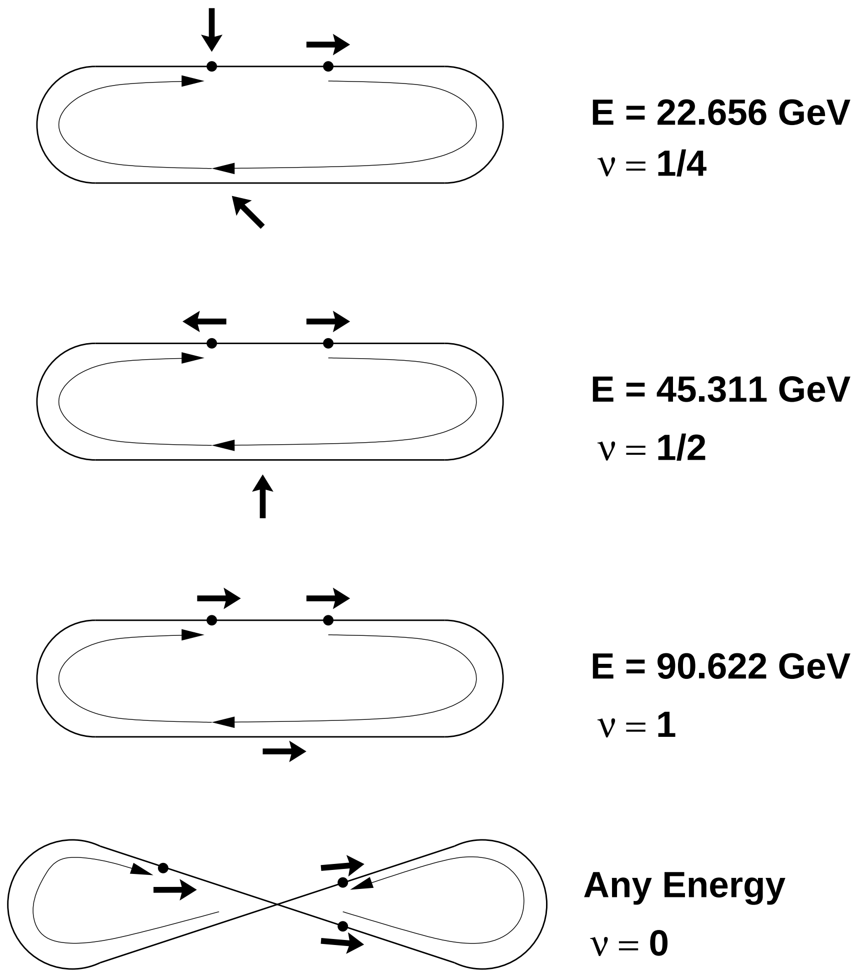


Fig. 1. *Several configurations of muon storage rings, showing the polarisation vector (thick arrows) of the muons for two consecutive turns.*

Maximising the polarisation for experiments requires one of the following situations, shown in figure 1.
–**half integer spin tune**: $E_\mu = 45.311 GeV, \nu = 0.5$, or higher energies, with $\nu = k+0.5$; this is the **favoured situation**, provides longitudinal polarisation with alternate sign turn after turn.
–**integer spin tune**: $E_\mu = 90.6223 GeV, \nu = 1$, or higher energies, with $\nu = k$; this provides the same longitudinal polarisation turn after turn. This seems like a very high energy, and requires an additional spin-flipping device (a single turn in a ring at $E_\mu = 45.311 GeV$ for example) if one wants to reverse the spin orientation.
– **zero spin tune**: the **bow-tie** configuration provides an interesting situation, since, provided the bow-tie is planar, there is no net spin precession. The price to pay is that the energy calibration by spin precession is no longer applicable. As in the the previous case, an additional spin-flipping device is required. It has been argued at the workshop that this configuration also allows both straight lines to go underground, thus providing two potential long

baselines orientations and avoiding the potential radiation problem of neutrino beams exiting the earth surface nearby.

In order to obtain longitudinal polarisation within 3% of the maximum, the energies in the first two cases must be exact to within $\pm 2 \times 10^{-4}$. Spin precession in this case also provides energy calibration to a much better precision, so this should not be an important difficulty.

3 Rings versus Bow-ties

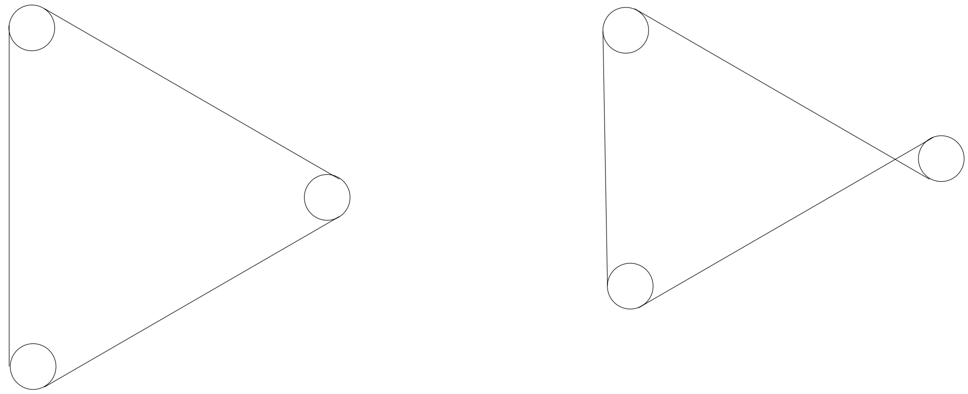
Bow-tie rings appear at first sight to be inefficient because the total amount of bending in the ring is larger than 180 degrees and a larger number of muon decays are lost. However if one considers the flux provided to neutrino oscillations experiments, bow-ties are particularily interesting if one wants to provide long baseline neutrino beams to two different directions. Scenarios where a muon storage ring situated at BNL, Fermilab or CERN would service, for example, both Gran Sasso Laboratory and the Soudan Mine, would lead to an angle of about 60 degrees between the two beam lines. To provide this with a ring geometry implies a rather long upward going straight section. The bow tie configuration reduces this to some extent, and, although the total fraction of muons decaying in straight sections is reduced, the fraction that is used in the long baseline beams can actually be increased. This is illustrated in figure 2. There is a large variety of bow-tie configurations, since in particular the up-going section can be split in two, providing two high quality short baseline beams.

4 Depolarisation due to energy spread

Secondly, depolarisation occurs. Because of energy spread, the spins of particles with different energies precess differently. There is no such depolarisation in a bow-tie configuration since the spin-tune is zero. If the ring is planar and polarisation is in its plane, the polarisation component outside of the plane will remain null. Will remain a longitudinal component $\mathcal{P}_L$ and a transverse component $\mathcal{P}_x$. As muons circulate in the ring, the polarisation precesses in the plane of the ring, so that, at turn T after injection:

$$[\mathcal{P}_L + i\mathcal{P}_x](T) \propto \int e^{i2\pi\nu T} S(\nu) d\nu$$

where $S(\nu)$ is the distribution of spin-tunes (i.e. of energies) within the bunch of muons.



length of short straight section: 375 m

long straight/total = 0.341
all decay/total = 0.857

length of short straight section: 289 m

long straight/total = 0.364
all decay/total = 0.822

length of long straights 375 meters, angle between long straights 2x30 degrees
beam energy 45 GeV, average field in arcs 6 Tesla.

Fig. 2. *Symmetric rings with 60º opening angle*

Depolarisation will be very fast if the energy spread is large and if no RF system is foreseen to keep the muons bunched. If the muon storage ring is equipped with a RF system that imposes synchrotron oscillations with a tune Q_s, the above formula is modified:

$$[\mathcal{P}_L + i\mathcal{P}_x]\,(T) \propto \int e^{i2\pi \sum_{n=1,T} \nu_0 + (\nu-\nu_0)\cos(2\pi n Q_s)} S(\nu) d\nu$$

where ν_0 is the nominal spin tune, and $S(\nu)$ is the distribution of spin-tunes at the time the muons are injected in the ring; (one recovers the previous formula for $Q_s = 0$). The effect is to reduce depolarisation dramatically, as can be seen in table 1, which also indicates the value of Q_s which is necessary to cancel depolarisation due to energy spread, $Q_s \approx \sigma(E)/E$.

The muon polarisation can be monitored by momentum analysis of the decay electrons, as discussed in ref. [4], where a polarimeter layout is described. The spin precession provides means of high precision energy calibration [5]. Figure 3 shows the turn-by-turn oscillations of decay electron intensity during a muon fill. In the case where there is no RF, the depolarisation due to energy spread is dramatic, but this fill alone would be sufficient to measure the beam energy and its spread, with a precision of 2×10^{-4} of the beam energy.

The precision with which the polarisation could be monitored is difficult to assess without having made a simulation of the polarimeter operation with realistic beam parameters and related uncertainties. It is probably of the order or better than 1% of the polarisation it self, but this, as well as the measurements of the total muon beam intensity and angular divergence, requires more

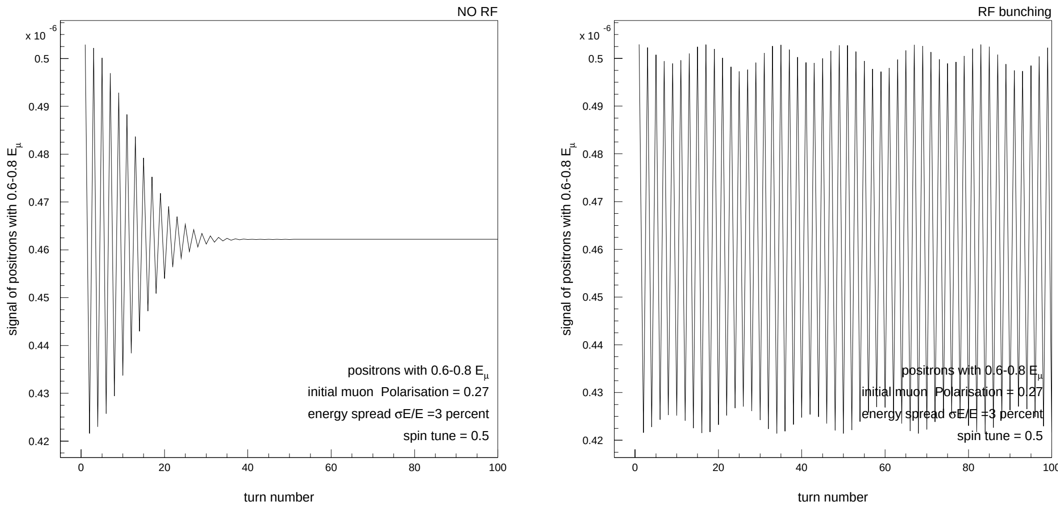


Fig. 3. *Oscillation with turn number in a fill of the number of electrons in the energy range 0.6-0.8* E_μ *(normalised to the total number of muon decays during the given turn). The oscillation amplitude is a measure of the beam polarisation, its frequency a measure of the beam energy, and, if their is no RF bunching, its decrease with time is a measure of energy spread. The muon lifetime corresponds here to 300 turns. The beam energy is* $E_\mu = 45.311 GeV$*, the energy spread is* 3×10^{-2}*. On the left, there is no bunching RF in the muon storage ring, on the right there is RF bunching with* $Q_s = 0.03$

detailed studies.

	effective polarisation $\sqrt{<\mathcal{P}^2>}$						
		RF bunching with $Q_s =$					
$\sigma(E)/E$	no RF bunching	0.001	0.002	0.005	0.01	0.03	0.1
10^{-3}	0.203	0.255	0.266	0.269	0.270	0.270	0.270
2×10^{-3}	0.162	0.221	0.255	0.267	0.269	0.270	0.270
5×10^{-3}	0.111	0.140	0.198	0.254	0.266	0.269	0.270
10^{-2}	0.081	0.100	0.134	0.210	0.254	0.268	0.270
3×10^{-2}	0.048	0.059	0.077	0.120	0.179	0.237	0.268

Table 1
Effective polarisation for muon fills at $E_\mu = 45.311 GeV, \nu = 0.5$*, for various conditions of energy spread without and with RF bunching for different values of* Q_s.

5 Neutrino fluxes and muon polarization

Now that we have established the conditions under which longitudinal polarisation could be obtained and monitored, one can ask oneself: what will it be useful for?

Neutrino spectra with different polarisation are given by the following equations valid for μ^+ decays in the muon center-of-mass (reverse polarisations for μ^-):
– for the ν_μ:

$$\frac{d^2N}{dxd\cos\theta} = Nx^2[(3-2x) - \mathcal{P}(1-2x)\cos\theta]$$

– for the ν_e:

$$\frac{d^2N}{dxd\cos\theta} = 6Nx^2[(1-x) - \mathcal{P}(1-x)\cos\theta]$$

where θ is the decay angle in the muon centre-of-mass, $\mathcal{P}$ the muon longitudinal polarisation, and $x = 2E_\nu/m_\mu$.

In a long base line experiment, one is at extremely small angles so that $\cos\theta = 1$. In this case, the ν_e component of the beam is completely extinct for $\mathcal{P} = +1$ [6]. This is due to spin conservation in the decay, a right handed muon cannot decay at zero angle into a left-handed ν_e.

Event numbers can be readily obtained by multiplying by the cross-section assuming a 10m radius detector 20 m long situated at 730 km. They can be seen in figure 4. Since the neutrino and anti-neutrino cross-sections are in the ratio 1/0.45 negative muons provide enrichment in ν_μ and positive ones in ν_e.

It is clear from figure 4 that the combination of muon sign and polarisation allows large variations in the composition of the beam, in a nicely controlled way. What use can be made of it in the context of neutrino oscillation studies and what gain does polarisation bring, has not been evaluated quantitatively yet. As a possible clue, the fraction of ν_e events among all CC events is given in figure 5 as function of muon polarisation and for the two muon signs. By switching from negative muons with 50% negative polarisation to positive muons with 50% negative polarisation, one can change the ratio of CC ν_e to CC ν_μ by a factor 20, this ratio is only 5 in absence of polarisation. This must be useful.

One possible application would be the observation of $\nu_\mu \to \nu_e$ oscillations, where the signal consists of appearance of Charged Current ν_e events, with a high energy electron. It is assumed that the detector could not distinguish the

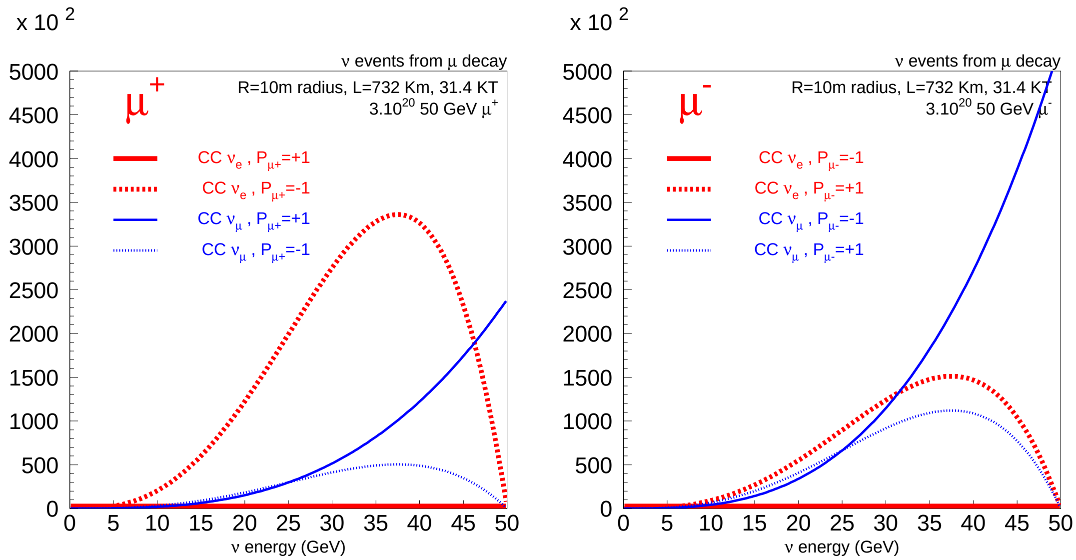


Fig. 4. *Event numbers for a detector of density 5, 10 m radius, 20 m long, situated 732 km away from the muon storage ring, for* $\mu^+ \rightarrow e^+\nu_e\bar{\nu_\mu}$ *(left) and* $\mu^- \rightarrow e^-\bar{\nu_e}\nu_\mu$*(right) beams of 50 GeV. Full lines show the spectra for "natural helicity"* $\mathcal{P} = +1$ *for* μ^+ *, and dashed ones for the opposite. The* $CC\nu_e$ *for* μ^+ *with* $\mathcal{P} = +1$ *and* $CC\nu_e$ *for* μ^- *with* $\mathcal{P} = -1$ *are not visible, because the flux is almost exactly zero. Vertical axis gives event numbers per bin of 250 MeV. This assumes no muon beam angular divergence and no beam energy spread.*

sign of the electrons, but that it could recognise, at least statistically events with a large electromagnetic component. The spectrum of these events is that of the ν_μ component of the beam, which is quite different from the original ν_e one. The high energy part of the spectrum is quite indicated to do that, since in addition this would suppress the background from neutral current events. The fraction of ν_e events among all CC events for the high energy part of the spectrum is also shown figure 5.

6 Effect of beam divergence

Because polarisation effects happen in the forward direction, they are quite sensitive to the beam divergence. The spectra were calculated for the same detector but now folding a muon beam divergence. The result is shown for 45.311 GeV muons on figure 6. It is clear that beam divergence results in a loss of events, and in a sizeable distortion of the spectra and of their muon polarisation dependence. A beam divergence not larger than $\sigma\theta_x = \sigma\theta_y = 0.05 m_\mu/E_\mu$ seems to be desirable, (the exact figure will have to be calculated precisely) if one want to avoid a large sensitivity of physics results upon the experimental determination of the muon beam parameters.

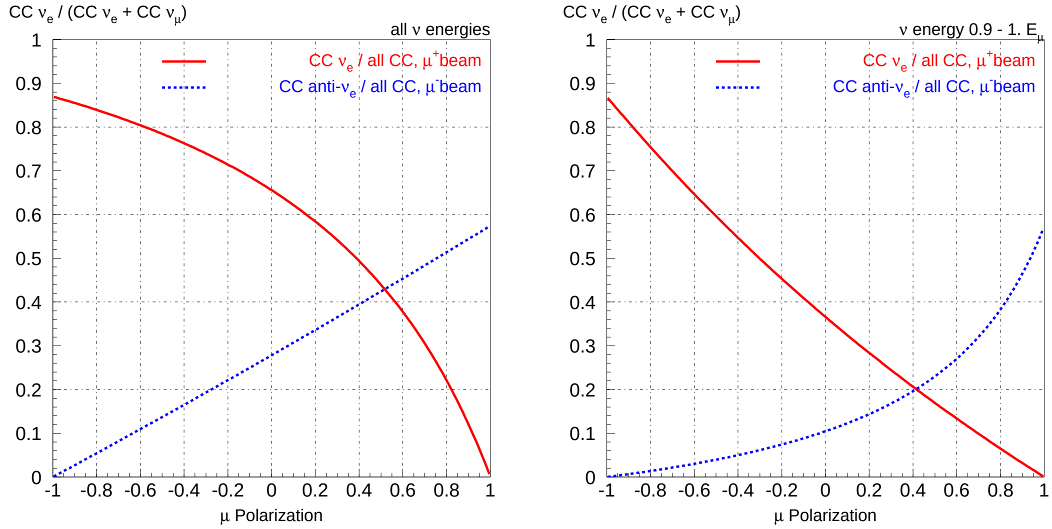


Fig. 5. *Fraction of ν_e events among all CC events as function of muon polarisation and for the two muon signs. On the left: all energies; on the right: the high energy end of the spectrum.*

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References

[1] R. Palmer, these proceedings.

[2] A. Blondel and J.M. Jowett, Wigglers for polarization, in *Polarization at LEP*, CERN88-06, (1988) Vol2 p. 216.

[3] C. Ackenbrandt et al., "Status of Muon Collider Research and Development and future plans" The muon collider collaboration, BNL-65623, Fermilab-PUB-98/179, LBNL-41935, http://www.cap.bnl.gov/mumu/status_report.html.

[4] A. Blondel, in *Prospective study of muon storage rings at CERN*, CERN 98-02, ECFA 99-197 (1999) p. 51-54.

[5] R. Raja and A. Tollestrup, Calibrating the energy of a 50×50 GeV Muon Collider using $g-2$ spin precession, Phys. Rev. D58, (1998)13005; http://xxx.lanl.gov/ps/hep-ex/9801004

[6] S. Geer, Phys. Rev. **D57** (1998) 6989.

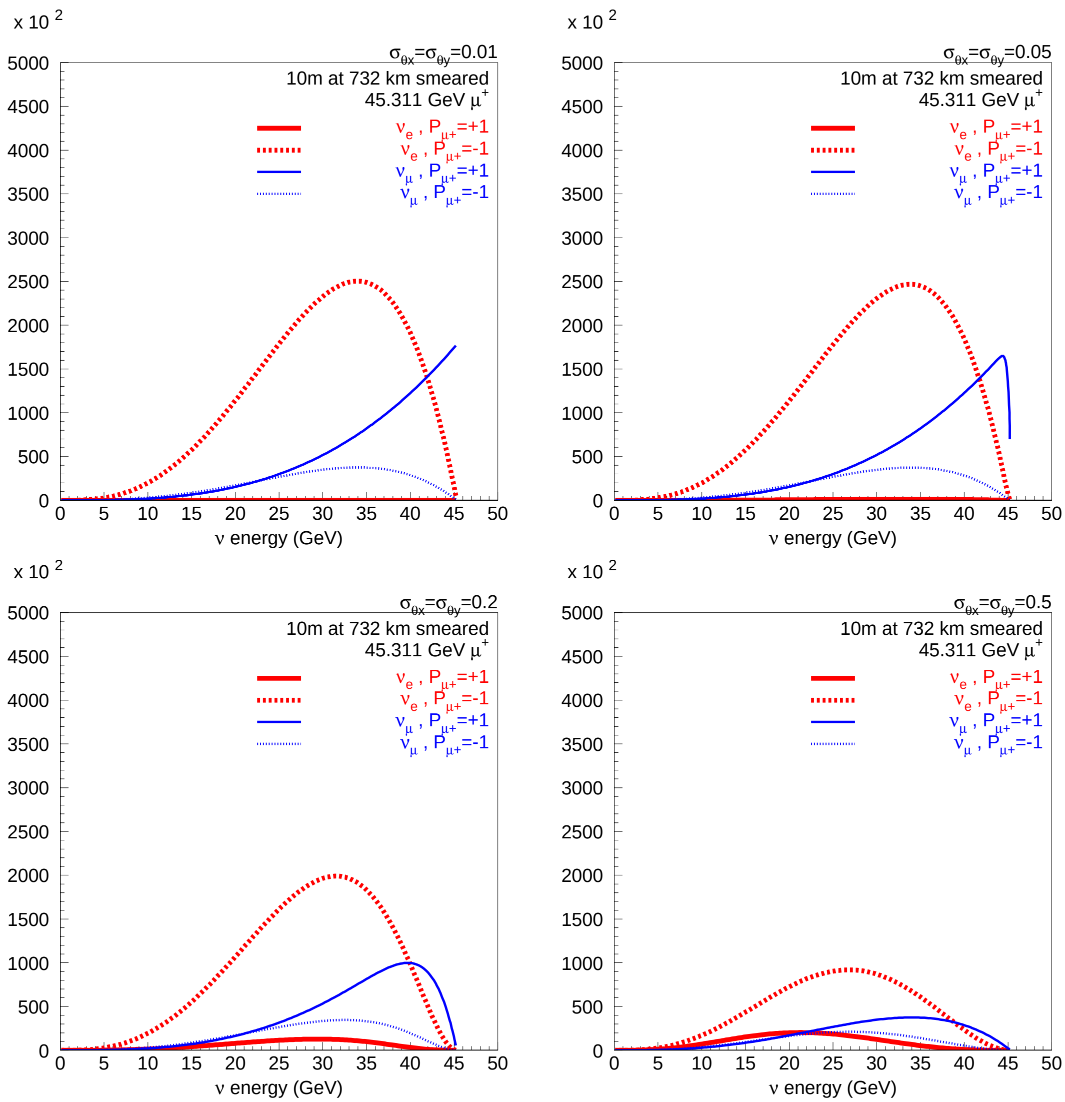

Fig. 6.
Neutrino event spectra for different beam divergences. upper left: $\sigma\theta_x = \sigma\theta_y = 0.01\ m_\mu/E_\mu$
upper right: $\sigma\theta_x = \sigma\theta_y = 0.05\ m_\mu/E_\mu$
lower left: $\sigma\theta_x = \sigma\theta_y = 0.2\ m_\mu/E_\mu$
lower right: $\sigma\theta_x = \sigma\theta_y = 0.5\ m_\mu/E_\mu$
It is clear that beam divergence results in a loss of events, and in a sizeable distorsion of the spectra and of their muon polarisation dependence.